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STABILITY OF A VISCOUS LIQUID JET IN A HIGH-FREQUENCY ALTERNATING ELECTRIC FIELD

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The stability of a liquid electrolyte placed in a tangential electric field oscillating harmonically at high frequency is considered assuming that the liquid is viscous and Newtonian. It is shown that, if the Peclet number calculated from the thickness of the Debye layer is small, the problem can be solved separately for the electrodynamic part of the problem in the Debye layer and for the hydrodynamic part of the problem in the jet. The linear stability of the trivial solution of the problem is investigated. A dispersion relation is derived and used to study the effect of the amplitude and frequency of electric field oscillations on the stability of the jet. It is shown that the presence of the external oscillating field has a stabilizing effect on the jet. The basic stability regimes as functions of the control parameters of the problem and bifurcation changes in the regimes are investigated.

Key words: *electrohydrodynamics, electrolyte, linear stability, viscous liquid, microjet.*

Introduction. The problem of the behavior of a capillary jet in an external electric field is a classical problem of electrohydrodynamics and has numerous applications, in particular, as a liquid spraying method (printers, car carburetors, fuel spraying, etc.). The case of a DC external electric field and a charge on the jet surface has been studied in many papers (see, for example, [1–5]), which have shown that the presence of a charge on the free surface leads to destabilization of the jet with respect to long-wave perturbations, and the presence of an external tangential field, in contrast, stabilizes the jet. As is known, capillary jets are unstable and break up into droplets [2, 6]. Experimental investigation of liquid jets and droplets acted upon by an electric field was pioneered in [7]. There are a large number of models describing various features of the processes studied. A review of the various models currently available is given in [8].

The behavior of a jet depends strongly on whether the working jet liquid is a dielectric, conductor, or electrolyte. At present, electrolytes are the least studied, although they have been investigated in experiments [7] and are often used in practice for electrospraying.

In some experimental paper, in particular, [9], it is proposed to use an alternating electric field of high frequency instead of a constant field. Advantages of this external force field are the presence of a new control parameter, i.e., the oscillation frequency; electroneutrality of the liquid droplets formed during spraying; the absence of undesirable chemical reactions at high oscillation frequencies (above 10 kHz) because the oscillation period is much shorter than the characteristic reaction time. The purpose of the present work is to develop a model to describe phenomena of this type.

We note that, initially, the behavior of liquid droplets and jets in an external electric field was described by static models or models assuming an ideal liquid. This approach is permissible because the primary instability is caused by normal stresses on the interface. Nevertheless, later papers [10, 11] reported some contradictions resulting from the ideal liquid assumption and discrepancies between theoretical results and experimental data. For example, studying the collapse of liquid jets, Eggers [10] notes that a thin boundary layer near the neck that arises at an arbitrarily low viscosity plays a key role in the collapse. Cherney [11] gives an asymptotic solution of the problem

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of emission of a liquid jet from a Taylor cone and shows that the jet is formed from a thin boundary layer near the cone–gas interface. Therefore, the inviscid liquid model appears to be unsuitable for describing the behavior of the liquid jet issuing from a Taylor cone at large values of the potential difference.

The present paper studies the stability of a viscous capillary liquid jet of a cylindrical shape in a tangential electric field oscillating at a frequency $\tilde{\omega}$ of order $\tilde{\varkappa}/\tilde{\varepsilon}$ ($\tilde{\varkappa}$ is the electrical conductivity and $\tilde{\varepsilon}$ is the dielectric permittivity of the liquid). The mechanical motion in the system induced by this field includes a slowly varying average component and a fast oscillatory component [12]. The resulting motion is their superposition, with the contribution of the oscillatory component tending to zero with increasing oscillation frequency [12].

By linearizing the systems near the trivial solution, we obtain the eigenvalue problem which depends on the wavenumber and linear growth coefficient of the perturbations and the main dimensionless parameters of the system. In the present paper, we study the dependence of the stability region on the specified parameters and the transitions between various stability regimes upon reaching critical values of the control parameter. It is shown that an increase in the oscillation amplitude of the external field leads to stabilization of the jet due to narrowing of the range of unstable wavelengths and due to a decrease in the maximum value of the linear growth coefficient. An increase in the oscillation frequency, in contrast, leads to insignificant destabilization of the jet. It is established that for fixed physicochemical properties of the liquid, the field oscillation frequency has two critical values that correspond to changes in the stability regime.

1. Formulation of the Problem. We consider a viscous liquid electrolyte jet placed in an external tangential electric field. The processes occurring in the liquid phase are described by two transfer equations for negative and positive ions, the Poisson equation for the electric field potential, the hydrodynamic Navier–Stokes equations, and the continuity equation. The liquid is a simple binary electrolyte: $z^+ = -z^- = 1$. The diffusion coefficients of negative and positive ions are set equal to each other: $\tilde{D}^+ = \tilde{D}^- = \tilde{D}$. The complete system of equations is written as

$$\frac{\partial \tilde{c}^{\pm}}{\partial \tilde{t}} + \tilde{\boldsymbol{U}} \cdot \tilde{\nabla} \tilde{c}^{\pm} = \tilde{D}\Big(\pm \frac{\tilde{F}}{\tilde{R}\tilde{T}} \tilde{\nabla} \cdot \Big(\tilde{c}^{\pm} \tilde{\nabla} \tilde{\Phi} \Big) + \tilde{\nabla}^2 \tilde{c}^{\pm} \Big),$$

$$\tilde{\rho}\Big(\frac{\partial \tilde{\boldsymbol{U}}}{\partial \tilde{t}} + \Big(\tilde{\boldsymbol{U}} \cdot \tilde{\nabla} \Big) \tilde{\boldsymbol{U}} \Big) = -\tilde{\nabla}\tilde{p} + \tilde{\mu}\tilde{\nabla}^2 \tilde{\boldsymbol{U}} + \tilde{F}(\tilde{c}^- - \tilde{c}^+)\tilde{\nabla}\tilde{\Phi}, \tag{1}$$

$$\tilde{\nabla} \cdot \tilde{\boldsymbol{U}} = 0, \qquad \tilde{\nabla}^2 \tilde{\Phi} = \tilde{F}\,(\tilde{c}^- - \tilde{c}^+)/\tilde{\varepsilon}.$$

Here $\tilde{c}^+$ and $\tilde{c}^-$ are the mole concentrations of cations and anions, respectively, $\tilde{\boldsymbol{U}}$ is the liquid velocity vector, $\tilde{D}$ is the diffusivity of ions, $\tilde{F}$ is the Faraday number, $\tilde{R}$ is the universal gas constant, $\tilde{T}$ [K] is the temperature, $\tilde{\Phi}$ is the electric field potential, $\tilde{\varepsilon}$ is the absolute dielectric permittivity of the liquid, $\tilde{p}$ is the pressure, $\tilde{\mu}$ is the dynamic viscosity, and $\tilde{\rho}$ is the density of the liquid; the dimensionless quantities are marked by the tilde. Away from the interface, the solution is considered electroneutral:

$$\tilde{c}^{\pm} = \tilde{c}_{\infty}. \tag{2}$$

The gas surrounding the jet is considered a dielectric. The electric field potential in the gas phase satisfies the Laplace equation

$$\tilde{\nabla}^2 \bar{\tilde{\Phi}} = 0. \tag{3}$$

Here and below, the quantities corresponding to the external problem for the gas are marked by the bar.

In the present work, the most important case of axisymmetric perturbations of the jet is investigated. The following cylindrical coordinates are introduced: $\tilde{x}$ is the coordinate along the axis of the unperturbed jet and $\tilde{y}$ is the radial coordinate. To describe the process near the free boundary of the liquid phase, it is reasonable to use the coordinates $\tilde{n}$ and $\tilde{\tau}$ [$\tilde{n}$ is the coordinate along the unit outward normal $\boldsymbol{n}$ to the interface; $\tilde{\tau}$ is the coordinate along the unit tangent $\boldsymbol{\tau}$ in the plane $(\tilde{x}, \tilde{y})$ (Fig. 1)]. The boundary conditions on the interface are given below.

The gas is assumed to be nonconducting; therefore, the flux of negative and positive ions through the free surface is equal to zero:

$$\tilde{n} = 0: \qquad \pm \frac{\tilde{c}^{\pm}\tilde{F}}{\tilde{R}\tilde{T}} \frac{\partial \tilde{\Phi}}{\partial \tilde{n}} + \frac{\partial \tilde{c}^{\pm}}{\partial \tilde{n}} = 0. \tag{4}$$

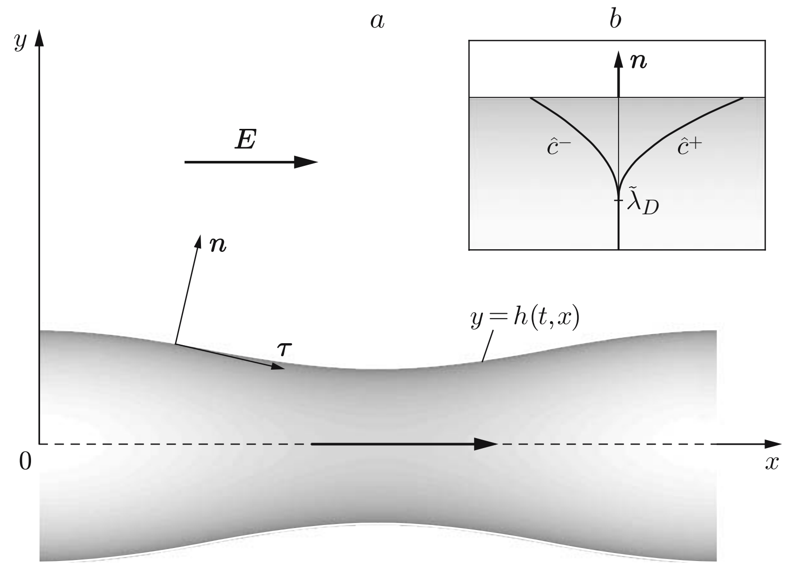

Fig. 1. Diagram of the problem: (a) coordinate systems in the problem of liquid jet motion under the action of external tangential electric field E_∞; (b) boundary layer near the free boundary of the liquid.

The potential across the interface is continuous:

$$\tilde{n} = 0: \qquad \tilde{\Phi} = \tilde{\tilde{\Phi}}. \tag{5}$$

Because of the difference in dielectric permittivity between the media, the derivative of the potential along the normal undergoes a discontinuity:

$$\tilde{\varepsilon}\,\frac{\partial \tilde{\Phi}}{\partial \tilde{n}} = \tilde{\tilde{\varepsilon}}\,\frac{\partial \tilde{\tilde{\Phi}}}{\partial \tilde{n}} \tag{6}$$

($\tilde{\tilde{\varepsilon}}$ is the dielectric permittivity of the gas surrounding the jet). The balance conditions for the normal and shear stresses, respectively, are written as

$$[\boldsymbol{n}\tilde{T}^F\boldsymbol{n}] + [\boldsymbol{n}\tilde{T}^E\boldsymbol{n}] = \tilde{\gamma}\tilde{K}, \qquad [\boldsymbol{n}\tilde{T}^F\boldsymbol{\tau}] + [\boldsymbol{n}\tilde{T}^E\boldsymbol{\tau}] = 0, \tag{7}$$

where square brackets denote a jump of the quantity across the interface, $\tilde{\gamma}$ is the surface tension, $\tilde{K}$ is the average curvature of the interface, $\tilde{T}^F$ is the hydrodynamic stress tensor, and $\tilde{T}^E$ is the Maxwell–Wagner electrical stress tensor, whose components in any orthogonal coordinate system are given by

$$\tilde{T}^E_{ij} = \tilde{\varepsilon}(-\tilde{E}^2\delta_{ij}/2 + \tilde{E}_i\tilde{E}_j).$$

In particular, in the coordinates $\tilde{n}$ and $\tilde{\tau}$, the normal electrical stress (electrical pressure) and the tangential electrical stress (electrical shift) are written as

$$\boldsymbol{n}\tilde{T}^E\boldsymbol{n} = \tilde{T}^E_{nn} = \tilde{\varepsilon}(\tilde{E}_n^2 - \tilde{E}_\tau^2)/2, \qquad \boldsymbol{n}\tilde{T}^E\boldsymbol{\tau} = \tilde{T}^E_{nn} = \tilde{\varepsilon}\tilde{E}_n\tilde{E}_\tau. \tag{8}$$

The kinematic condition is written as

$$\tilde{v} = \frac{\partial \tilde{h}}{\partial \tilde{t}} + \tilde{u}\,\frac{\partial \tilde{h}}{\partial \tilde{x}}, \tag{9}$$

where $\tilde{y} = \tilde{h}(\tilde{t}, \tilde{x})$ is the equation of the interface in cylindrical coordinates and $\tilde{u}$ and $\tilde{v}$ are the liquid velocity components $\tilde{x}$ and $\tilde{y}$, respectively.

The problem is closed by the boundary condition at infinity. The oscillations of the external field are considered harmonic:

$$\tilde{\tilde{\Phi}} = \tilde{\Phi}_\infty = -\tilde{x}\tilde{E}_\infty\,\mathrm{e}^{i\tilde{\omega}\tilde{t}} + \text{c. c.} \tag{10}$$

(c. c. denotes a complex-conjugate quaintly).

System (1)–(10) is reduced to dimensionless form by dividing the lengths, concentrations, electric potential, velocities, time, and pressure, respectively, by the quantities

$$\tilde r_0, \quad \tilde c_\infty, \quad \tilde\Phi_0 = \Big(\frac{\tilde\gamma \tilde r_0}{\tilde\varepsilon}\Big)^{1/2}, \quad \tilde U_0 = \Big(\frac{\tilde\gamma}{\tilde\rho \tilde r_0}\Big)^{1/2}, \quad \tilde t_0 = \frac{\tilde r_0}{\tilde U_0}, \quad \tilde p_0 = \tilde\rho \tilde U_0^2,$$

where $\tilde r_0$ is the radius of the unperturbed jet.

System (1) is written in dimensionless form

$$\mathrm{Pe}\Big(\frac{\partial c^\pm}{\partial t} + u\,\frac{\partial c^\pm}{\partial x} + v\,\frac{\partial c^\pm}{\partial y}\Big)$$

$$= \pm\Lambda\Big\{\frac{\partial}{\partial x}\Big(c^\pm\,\frac{\partial\Phi}{\partial x}\Big) + \frac{1}{y}\,\frac{\partial}{\partial y}\Big(yc^\pm\,\frac{\partial\Phi}{\partial y}\Big)\Big\} + \frac{\partial^2 c^\pm}{\partial x^2} + \frac{1}{y}\,\frac{\partial}{\partial y}\Big(y\,\frac{\partial c^\pm}{\partial y}\Big); \tag{11}$$

$$\frac{\partial u}{\partial t} + u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\Big\{\frac{\partial^2 u}{\partial x^2} + \frac{1}{y}\,\frac{\partial}{\partial y}\Big(y\,\frac{\partial u}{\partial y}\Big)\Big\} + \frac{1}{\varepsilon^2}\,(c^- - c^+)\,\frac{\partial\Phi}{\partial x},$$

$$\frac{\partial v}{\partial t} + u\,\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\Big\{\frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial y}\Big(\frac{1}{y}\,\frac{\partial}{\partial y}\,yv\Big)\Big\} + \frac{1}{\varepsilon^2}\,(c^- - c^+)\,\frac{\partial\Phi}{\partial y}; \tag{12}$$

$$\frac{\partial u}{\partial x} + \frac{1}{y}\,\frac{\partial}{\partial y}\,(yv) = 0; \tag{13}$$

$$\varepsilon^2\Big\{\frac{\partial^2\Phi}{\partial x^2} + \frac{1}{y}\,\frac{\partial}{\partial y}\Big(y\,\frac{\partial\Phi}{\partial y}\Big)\Big\} = c^- - c^+, \qquad y < h; \tag{14}$$

$$\frac{\partial^2\bar\Phi}{\partial x^2} + \frac{1}{y}\,\frac{\partial}{\partial y}\Big(y\,\frac{\partial\bar\Phi}{\partial y}\Big) = 0, \qquad y > h; \tag{15}$$

for $y = h$,

$$\pm\Lambda c^\pm\,\frac{\partial\Phi}{\partial n} + \frac{\partial c^\pm}{\partial n} = 0; \tag{16}$$

$$\Phi = \bar\Phi, \quad \delta\,\frac{\partial\Phi}{\partial n} = \frac{\partial\bar\Phi}{\partial n}, \quad [\boldsymbol{n}T^F\boldsymbol{n}] + [\boldsymbol{n}T^E\boldsymbol{n}] = K, \quad [\boldsymbol{n}T^F\boldsymbol{\tau}] + [\boldsymbol{n}T^E\boldsymbol{\tau}] = 0; \tag{17}$$

$$v = \frac{\partial h}{\partial t} + u\,\frac{\partial h}{\partial x}; \tag{18}$$

for $y = \infty$,

$$\bar\Phi = -xE_\infty\,\mathrm{e}^{i\omega t} + \text{c. c.} \tag{19}$$

Here the parameters of the system

$$\mathrm{Pe} = \frac{\tilde U_0 \tilde r_0}{\tilde D}, \quad \mathrm{Re} = \frac{\tilde\rho\tilde r_0\tilde U_0}{\tilde\mu}, \quad \varepsilon^2 = \frac{\tilde\lambda_D^2}{\tilde r_0^2} = \frac{\tilde\varepsilon\tilde\Phi_0}{\tilde F\tilde c_\infty\tilde r_0^2}, \quad \Lambda = \frac{\tilde F\tilde\Phi_0}{\tilde R\tilde T}, \quad \delta = \frac{\tilde\varepsilon}{\tilde{\bar\varepsilon}}$$

are dimensionless; $\tilde\lambda_D = (\tilde\varepsilon\tilde\Phi_0/\tilde F\tilde c_\infty)^{1/2}$ is the Debye layer thickness. Away from the interface, $c^\pm = 1$. Usually, in experiments, the parameter ε is varied in the range 10^{-6}–10^{-2}. The problem is solved under the assumption that $\varepsilon \ll 1$ and $\varepsilon^2\,\mathrm{Pe} \ll 1$. From Eq. (14), it follows that, in this case, a small parameter at the higher derivative appears which generates a boundary layer in the vicinity of the free surface of the liquid. Thus, in the limit $\varepsilon \to 0$, the problem splits into an external problem (away from the interface) and an internal problem (near the interface).

2. Internal Problem. Following [13], we consider the solution of system (11)–(19) in a thin Debye layer of thickness of order $O(\varepsilon)$. Next, it is shown that, with some additional assumptions in this region, it is possible to ignore hydrodynamic effects and solve the electrodynamic problem independently.

In the internal coordinates $\xi = x$ and $\eta = (y-h)/\varepsilon$, Eqs. (11) and (14) are written as

$$\varepsilon^2\,\mathrm{Pe}\,\frac{\partial c^\pm}{\partial t} + \varepsilon^2\,\mathrm{Pe}\Big(u\,\frac{\partial c^\pm}{\partial \xi} + v\,\frac{\partial c^\pm}{\partial \eta}\Big) = \pm\Lambda\Big\{\varepsilon^2\frac{\partial}{\partial\xi}\Big(c^\pm\,\frac{\partial\Phi}{\partial\xi}\Big)$$

$$+\,\frac{1}{h+\varepsilon\eta}\,\frac{\partial}{\partial\eta}\Big((h+\varepsilon\eta)c^\pm\,\frac{\partial\Phi}{\partial\eta}\Big)\Big\} + \varepsilon^2\frac{\partial^2 c^\pm}{\partial\xi^2} + \frac{1}{h+\varepsilon\eta}\,\frac{\partial}{\partial\eta}\Big((h+\varepsilon\eta)\,\frac{\partial c^\pm}{\partial\eta}\Big); \tag{20}$$

$$\varepsilon^2\frac{\partial^2\Phi}{\partial\xi^2} + \frac{1}{h+\varepsilon\eta}\,\frac{\partial}{\partial\eta}\Big((h+\varepsilon\eta)\,\frac{\partial\Phi}{\partial\eta}\Big) = c^- - c^+. \tag{21}$$

Equations (20) and (21) are supplemented by dimensionless boundary conditions (2) and (16) written in the internal variables:

$$\eta\to-\infty:\qquad c^\pm\to 1,$$

$$\eta = 0:\qquad \pm\Lambda c^\pm\,\frac{\partial\Phi}{\partial\eta} + \frac{\partial c^\pm}{\partial\eta} = 0.$$

The main assumptions of this work are the following:

$$\varepsilon\ll 1,\qquad \varepsilon^2\,\mathrm{Pe}\ll 1,\qquad \Omega\equiv\Delta/(2\Lambda) = O(1) \tag{22}$$

($\Delta = \varepsilon^2\omega\,\mathrm{Pe}$ and $\omega = \tilde\omega\tilde t_0$ is the dimensionless frequency of electric field oscillations). The first assumption allows the interface in the internal coordinates to be considered locally flat. By virtue of the second assumption, the convective terms in Eq. (20) can be ignored and, hence, the electrodynamic part of the problem does not contain unknown liquid velocities. The third assumption actually specifies the allowable frequency range: the dimensionless oscillation frequency $\tilde\omega$ should be of order $\tilde\varkappa/\tilde\varepsilon$, where $\tilde\varkappa = 2\tilde F^2\tilde D\tilde c_\infty/(\tilde R\tilde T)$ is the electrical conductivity of the liquid. Under the assumptions made above, the quantity $\tilde\varkappa/\tilde\varepsilon$ has the meaning of the order of the frequency in the range 10^3–10^7 kHz [14] if the ion concentration $\tilde c_\infty$ varies in the range 10^{-3}–10 mole/m^3.

Thus, in a narrow Debye layer in a certain frequency range, the total problem is divided into two problems, and the electrodynamic problem can be solved earlier. For $\varepsilon\to 0$ and $\varepsilon^2\,\mathrm{Pe}\to 0$, Eqs. (20) and (21) have the form

$$\varepsilon^2\,\mathrm{Pe}\,\frac{\partial c^\pm}{\partial t} = \pm\Lambda\frac{\partial}{\partial\eta}\Big(c^\pm\,\frac{\partial\Phi}{\partial\eta}\Big) + \frac{\partial^2 c^\pm}{\partial\eta^2},\qquad \frac{\partial^2\Phi}{\partial\eta^2} = c^- - c^+. \tag{23}$$

In this case, the time derivative should not be ignored by virtue of the third condition in (21), according to which the oscillation frequency of the external electric field is high. To solve system (23), we use the Debye approximation

$$c^\pm = 1 + \hat c^\pm\,\mathrm{e}^{i\omega t} + \text{c. c.},\qquad \hat c^\pm\ll 1,\qquad \Phi = \hat\Phi^\pm\,\mathrm{e}^{i\omega t} + \text{c. c.} \tag{24}$$

Introducing the space charge distribution density $\rho = c^+ - c^-$ and $\rho = \hat\rho\,\mathrm{e}^{i\omega t} + $c. c., substituting (24) into Eq. (23), and linearizing the result, we obtain the following linear system of ordinary differential equations:

$$\frac{d^2\hat\rho}{d\eta^2} - (2\Lambda + i\Delta)\hat\rho = 0,\qquad \frac{d^2\hat\Phi}{d\eta^2} = -\hat\rho;$$

$$\eta = 0:\qquad 2\Lambda\,\frac{d\hat\Phi}{d\eta} + \frac{d\hat\rho}{d\eta} = 0,\quad \hat\Phi = \hat{\bar\Phi};$$

$$\eta = -\infty:\qquad \hat\rho = 0$$

(the potential $\hat{\bar\Phi}$ is taken from the solution of the problem for the gas). From this system of equations, we obtain the relationship between the complex amplitudes of the surface charge density $\hat\sigma$ and electric field intensity:

$$\hat\sigma = \int\limits_{-\infty}^{0}\hat\rho\,d\eta = \frac{i}{\Omega}\,\frac{d\hat\Phi}{d\eta}\Big|_{\eta=-\infty},$$

and the jump of the potential amplitude in the internal zone

$$\hat\Phi\Big|_{\eta=-\infty} - \hat{\bar\Phi} = \frac{i}{\Omega(2\Lambda+i\Delta)^{1/2}}\,\frac{d\hat\Phi}{d\eta}\Big|_{\eta=-\infty}. \tag{25}$$

In the external coordinates, the surface charge density is expressed in dimensionless form

$$\tilde{\sigma} = \frac{i\tilde{\varkappa}}{\tilde{\omega}} \frac{\partial \tilde{\hat{\Phi}}}{\partial \tilde{n}}\Big|_{\tilde{n} \ll -\varepsilon} \mathrm{e}^{i\tilde{\omega}\tilde{t}} + \text{c. c.} \tag{26}$$

3. External Problem. The passage to the limit $\varepsilon \to 0$ in Eq. (14) is equivalent to the assumption that the charge is located entirely on the interface and, hence, produces additional stress. The expression on the right side of formula (25) in the external coordinates has order $O(\varepsilon/\sqrt{\Lambda})$; therefore, for $\varepsilon \to 0$, condition (5) can be left without changes. In view of expressions (26), for the surface charge density, condition (6) should be replaced by the condition

$$\tilde{n} = 0: \qquad \tilde{\Phi} = \tilde{\bar{\Phi}}, \quad \tilde{\varepsilon} \frac{\partial \tilde{\Phi}}{\partial \tilde{n}} = \tilde{\bar{\varepsilon}} \frac{\partial \tilde{\bar{\Phi}}}{\partial \tilde{n}} + \tilde{\sigma}. \tag{27}$$

Thus, it is necessary to specify only boundary conditions (17) on the interface (here square brackets denote the difference between the values of the gas and liquid parameters outside the Debye layer). In view of (27), from (8) we obtain

$$\begin{gathered} [\boldsymbol{n}\tilde{T}^E\boldsymbol{n}] = \frac{\tilde{\varepsilon} - \bar{\varepsilon}}{2} \Big\{ \frac{\tilde{\varepsilon}}{\bar{\varepsilon}} \Big(\frac{\partial \tilde{\Phi}}{\partial \tilde{n}} \Big)^2 + \Big(\frac{\partial \tilde{\Phi}}{\partial \tilde{\tau}} \Big)^2 \Big\} - \frac{\tilde{\varepsilon}}{\bar{\varepsilon}} \frac{\partial \tilde{\Phi}}{\partial \tilde{n}} \tilde{\sigma} + \frac{1}{2\bar{\varepsilon}} \tilde{\sigma}^2, \\ [\boldsymbol{n}\tilde{T}^E\boldsymbol{\tau}] = -\frac{\partial \tilde{\Phi}}{\partial \tilde{\tau}} \tilde{\sigma}. \end{gathered} \tag{28}$$

By virtue of the electroneutrality condition, terms of order $O(1/\varepsilon^2)$ in Eqs. (12) containing the Coulomb force are absent in the external problem. The Poisson equation (14) becomes the Laplace equation for the potential inside the jet. Thus, for $\varepsilon \to 0$, the external problem has only the hydrodynamic part. Substituting dimensionless expressions (26) and (28) into the boundary conditions of system (11)–(19) and passing to the limit $\varepsilon \to 0$, we obtain the external problem in the form

$$\begin{gathered} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}} \Big\{ \frac{\partial^2 u}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \Big(y \frac{\partial u}{\partial y} \Big) \Big\}, \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}} \Big\{ \frac{\partial^2 v}{\partial x^2} + \frac{\partial}{\partial y} \Big(\frac{1}{y} \frac{\partial}{\partial y} yv \Big) \Big\}, \\ \frac{\partial u}{\partial x} + \frac{1}{y} \frac{\partial}{\partial y} (yv) = 0; \end{gathered} \tag{29}$$

$$\begin{gathered} \frac{\partial^2 \Phi}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \Big(y \frac{\partial \Phi}{\partial y} \Big) = 0, \qquad y < h, \\ \frac{\partial^2 \bar{\Phi}}{\partial x^2} + \frac{1}{y} \frac{\partial}{\partial y} \Big(y \frac{\partial \bar{\Phi}}{\partial y} \Big) = 0, \qquad y > h; \end{gathered} \tag{30}$$

for $y = h$,

$$\Phi = \bar{\Phi}, \qquad \delta \frac{\partial \Phi}{\partial n} = \frac{\partial \bar{\Phi}}{\partial n} + \sigma; \tag{31}$$

$$\begin{gathered} p - \frac{1}{\mathrm{Re}} (\tau_{xx} n_x^2 + 2\tau_{xy} n_x n_y + \tau_{yy} n_y^2) + \frac{\delta - 1}{2\delta} \Big\{ \delta \Big(\frac{\partial \Phi}{\partial n} \Big)^2 + \Big(\frac{\partial \Phi}{\partial \tau} \Big)^2 \Big\} - \delta\sigma \frac{\partial \Phi}{\partial n} + \frac{1}{2} \sigma^2 = K, \\ \frac{1}{\mathrm{Re}} \{ (\tau_{xx} - \tau_{yy}) n_x n_y + \tau_{xy} (n_y^2 - n_x^2) \} + \sigma \frac{\partial \Phi}{\partial \tau} = 0; \end{gathered} \tag{32}$$

$$v = \frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x}; \tag{33}$$

for $y = \infty$,

$$\bar{\Phi} = -x E_\infty \mathrm{e}^{i\omega t} + \text{c. c.} \tag{34}$$

Here n_x and n_y are the components of the normal vector $\boldsymbol{n}$:

$$n_x = -\frac{h_x}{(1+h_x^2)^{1/2}}, \qquad n_y = \frac{1}{(1+h_x^2)^{1/2}},$$

$$\frac{\partial}{\partial n} = n_x \frac{\partial}{\partial x} + n_y \frac{\partial}{\partial y}, \qquad \frac{\partial}{\partial \tau} = n_y \frac{\partial}{\partial x} - n_x \frac{\partial}{\partial y}, \qquad K = \frac{1}{h(1+h_x^2)^{1/2}} - \frac{h_{xx}}{(1+h_x^2)^{3/2}},$$

$$\tau_{xx} = 2\,\frac{\partial u}{\partial x}, \qquad \tau_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \qquad \tau_{yy} = 2\,\frac{\partial v}{\partial y}, \qquad \sigma = \frac{i\delta}{\Omega}\,\frac{\partial \hat\Phi}{\partial n}\Big|_{n=0}\,\mathrm{e}^{i\omega t} + \text{c. c.},$$

and the pressure in the gas $\tilde p_0 = 0$.

Returning to the initial formulation (11)–(19), we note that, although the averages of the electric potential and concentration are equal to zero, the Navier–Stokes equations, by virtue of their nonlinearity, include the electric force with a nonzero average. Thus, the liquid motion can be divided into the average and oscillatory motions. Following the classical averaging theory [12], it is easy to show that, to within small terms of higher order in frequency, the velocity field can be represented as

$$\boldsymbol{U}(t,x) = \bar{\boldsymbol{U}}_0(t,x) + \frac{1}{\omega}\,\tilde{\boldsymbol{U}}_1(t,x)\,\mathrm{e}^{i\omega t} + \text{c. c.},$$

where $\bar{\boldsymbol{U}}_0$ and $\tilde{\boldsymbol{U}}_1$ are the average and oscillatory velocity components. Below, the subscript 0 is omitted. Thus, for $\omega \to \infty$, the term containing $\tilde{\boldsymbol{U}}_1$ can be neglected.

The electrical quantities contain only the oscillatory component. The contribution to the average motion of the system comes from the nonlinear terms with nonzero averages:

$$\Big\langle\Big(\frac{\partial \Phi}{\partial \tau}\Big)^2\Big\rangle = 2\Big|\frac{\partial \hat\Phi}{\partial \tau}\Big|^2, \qquad \Big\langle\Big(\frac{\partial \Phi}{\partial n}\Big)^2\Big\rangle = 2\Big|\frac{\partial \hat\Phi}{\partial n}\Big|^2,$$

$$\langle\sigma^2\rangle = \frac{2\delta^2}{\Omega^2}\Big|\frac{\partial \hat\Phi}{\partial n}\Big|^2, \qquad \Big\langle\sigma\,\frac{\partial \Phi}{\partial \tau}\Big\rangle = -\frac{2\delta}{\Omega}\,\mathrm{Im}\Big(\frac{\partial \hat\Phi}{\partial n}\,\frac{\partial \hat\Phi}{\partial \tau}^{*}\Big), \qquad \Big\langle\sigma\,\frac{\partial \Phi}{\partial n}\Big\rangle = 0. \tag{35}$$

Here the asterisk denotes complex conjugation; angular brackets denote averaging over the fast time ωt:

$$\langle\,\cdot\,\rangle = \frac{1}{T}\int_0^T dt = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt.$$

The boundary conditions (32) averaged taking into account relations (35), become

$$p - \frac{1}{\mathrm{Re}}\,(\tau_{xx}n_x^2 + 2\tau_{xy}n_xn_y + \tau_{yy}n_y^2) + \Big(\frac{\delta^2}{\Omega^2} + \delta - 1\Big)\Big|\frac{\partial \hat\Phi}{\partial n}\Big|^2 + \Big(1 - \frac{1}{\delta}\Big)\Big|\frac{\partial \hat\Phi}{\partial \tau}\Big|^2 = K,$$

$$\frac{1}{\mathrm{Re}}\,\{(\tau_{xx} - \tau_{yy})n_xn_y + \tau_{xy}(n_y^2 - n_x^2)\} - \frac{2\delta}{\Omega}\,\mathrm{Im}\Big(\frac{\partial \hat\Phi}{\partial n}\,\frac{\partial \hat\Phi}{\partial \tau}^{*}\Big) = 0. \tag{36}$$

4. Linear Stability. Problem (19)–(34) has the trivial solution

$$h = 1, \quad u = v = 0, \quad p = 0, \quad \hat\Phi = \hat{\bar\Phi} = -E_\infty x.$$

The velocity components and the free boundary are subject to small perturbations:

$$u \sim \hat u(y)\,\mathrm{e}^{i\alpha x+\lambda t}, \quad v \sim \hat v(y)\,\mathrm{e}^{i\alpha x+\lambda t}, \quad p \sim \hat p(y)\,\mathrm{e}^{i\alpha x+\lambda t}, \quad h \sim 1 + \hat h\,\mathrm{e}^{i\alpha x+\lambda t}. \tag{37}$$

By virtue of Eqs. (30)–(34), the perturbations of the potential are parametric in time and are generated by the free-boundary perturbation (37). In particular, at the interface, to within terms of the higher order of smallness, we have

$$\hat\Phi\Big|_{y=h} = \hat\Phi\Big|_{y=1} + \frac{\partial \hat\Phi}{\partial y}\Big|_{y=1}\hat h\,\mathrm{e}^{i\alpha x+\lambda t} + \ldots .$$

Thus, the perturbations of the potential amplitudes should obey

$$\hat\Phi \sim -E_\infty x + \varphi(y)\hat h\,\mathrm{e}^{i\alpha x+\lambda t}, \qquad \hat{\bar\Phi} \sim -E_\infty x + \bar\varphi(y)\hat h\,\mathrm{e}^{i\alpha x+\lambda t}. \tag{38}$$

Let us consider the electrostatic part of the problem. Substitution of expressions (38) into Eqs. (30) yields the Bessel equations for the quantities φ and $\bar{\varphi}$. Requiring that the potential be regular for $y = 0$ and $y = \infty$, we obtain

$$\varphi(y) = C_1 I_0(\alpha y), \qquad \bar{\varphi}(y) = C_2 K_0(\alpha y), \tag{39}$$

where I_0 and K_0 are modified zero-order Bessel functions. To determine the unknown constants C_1 and C_2, we linearize conditions (31):

$$y = 1: \qquad \varphi = \bar{\varphi}, \quad \delta(1 - i/\Omega)(i\alpha E_\infty + \varphi') = i\alpha E_\infty + \bar{\varphi}'.$$

This implies that

$$C_1 = \frac{iE_\infty}{I_0}\,\Pi, \qquad C_2 = \frac{iE_\infty}{K_0}\,\Pi,$$
$$\Pi = \Pi(\delta, \alpha, \Omega) = \frac{1 - \delta(1 - i/\Omega)}{\delta(1 - i/\Omega) I_1(\alpha)/I_0(\alpha) - K_1(\alpha)/K_0(\alpha)} \tag{40}$$

($I_1 = I_0'$ and $K_1 = K_0'$ are modified first-order Bessel functions). From formulas (38)–(40), we obtain the solution of the electrostatic part of the of linear stability problem.

The obtained expressions for the complex amplitudes of the potential are used to linearize conditions (36):

$$\Bigl|\frac{\partial\hat{\Phi}}{\partial\tau}\Bigr|^2 \approx E_\infty^2(1 + 2\alpha\Pi_R\hat{h}\,\mathrm{e}^{i\alpha x + \lambda t}), \qquad \Bigl|\frac{\partial\hat{\Phi}}{\partial n}\Bigr|^2 \approx 0,$$
$$\mathrm{Im}\Bigl(\frac{\partial\hat{\Phi}}{\partial n}\,\frac{\partial\hat{\Phi}}{\partial\tau}^{*}\Bigr) \approx -iE_\infty^2\alpha\Pi_I\,\frac{I_1}{I_0}\,\hat{h}\,\mathrm{e}^{i\alpha x + \lambda t}\,. \tag{41}$$

Here $\Pi_R = (\Pi + \Pi^*)/2$ and $\Pi_I = (\Pi - \Pi^*)/(2i)$. Substituting expressions (37) and (41) into Eqs. (29) and conditions (33) and (36) and linearizing these equations and conditions, we obtain the eigenvalue problem

$$\lambda\hat{u} = -i\alpha\hat{p} + \frac{1}{\mathrm{Re}}\Bigl\{-\alpha^2\hat{u} + \frac{1}{y}\frac{\partial}{\partial y}\Bigl(y\,\frac{\partial\hat{u}}{\partial y}\Bigr)\Bigr\},$$
$$\lambda\hat{v} = -\hat{p}' + \frac{1}{\mathrm{Re}}\Bigl\{-\alpha^2\hat{v} + \frac{\partial}{\partial y}\Bigl(\frac{1}{y}\,\frac{\partial}{\partial y}\,y\hat{v}\Bigr)\Bigr\}, \tag{42}$$
$$i\alpha\hat{u} + \frac{1}{y}\,\frac{\partial}{\partial y}\,(y\hat{v}) = 0;$$

$$y = 1: \qquad \hat{p} - \frac{2}{\mathrm{Re}}\,\hat{v}' + (1 - \alpha^2)\hat{h} + \Bigl(1 - \frac{1}{\delta}\Bigr)2E_\infty^2\alpha\Pi_R\hat{h} = 0,$$
$$\frac{1}{\mathrm{Re}}\,(\hat{u}' + i\alpha\hat{v}) + \frac{2i\delta}{\Omega}\,E_\infty^2\alpha\Pi_I\,\frac{I_1}{I_0}\,\hat{h} = 0, \qquad \hat{v} = \lambda\hat{h}. \tag{43}$$

Introducing the Stokes stream function ψ

$$u = \frac{1}{y}\,\frac{\partial\psi}{\partial y}, \qquad v = -\frac{1}{y}\,\frac{\partial\psi}{\partial x},$$

we reduce system (42) to one equation. In terms of perturbations of the stream function ($\hat{\psi} = iy\hat{v}/\alpha$ and $\hat{\psi}' = y\hat{u}$), problem (42), (43) is written as

$$\Bigl(\frac{d^2}{dy^2} - \frac{1}{y}\,\frac{d}{dy} - \beta^2\Bigr)\Bigl(\frac{d^2}{dy^2} - \frac{1}{y}\,\frac{d}{dy} - \alpha^2\Bigr)\hat{\psi} = 0; \tag{44}$$

$$y = 1: \qquad -\lambda\hat{\psi}' + \frac{1}{\mathrm{Re}}\,(\hat{\psi}''' - \hat{\psi}'' + (1 - 3\alpha^2)\hat{\psi}' + 2\alpha^2\hat{\psi}) + i\alpha(1 - \alpha^2)\hat{h} + 2\Bigl(1 - \frac{1}{\delta}\Bigr)iE_\infty^2\alpha^2\Pi_R\hat{h} = 0,$$
$$\frac{1}{\mathrm{Re}}\,(\hat{\psi}'' - \hat{\psi}' + \alpha^2\hat{\psi}) + \frac{2i\delta}{\Omega}\,E_\infty^2\alpha\Pi_I\,\frac{I_1}{I_0}\hat{h} = 0, \quad \alpha\hat{\psi} = i\lambda\hat{h}, \tag{45}$$

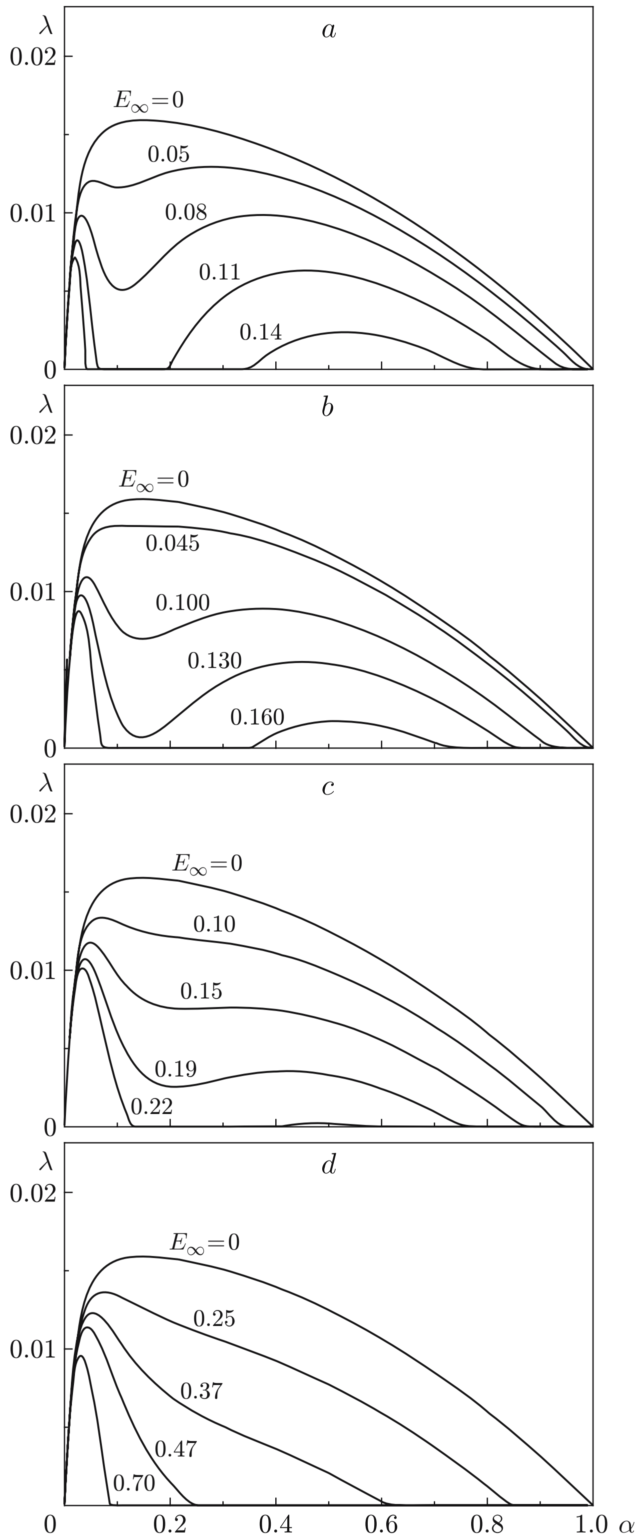

Fig. 2

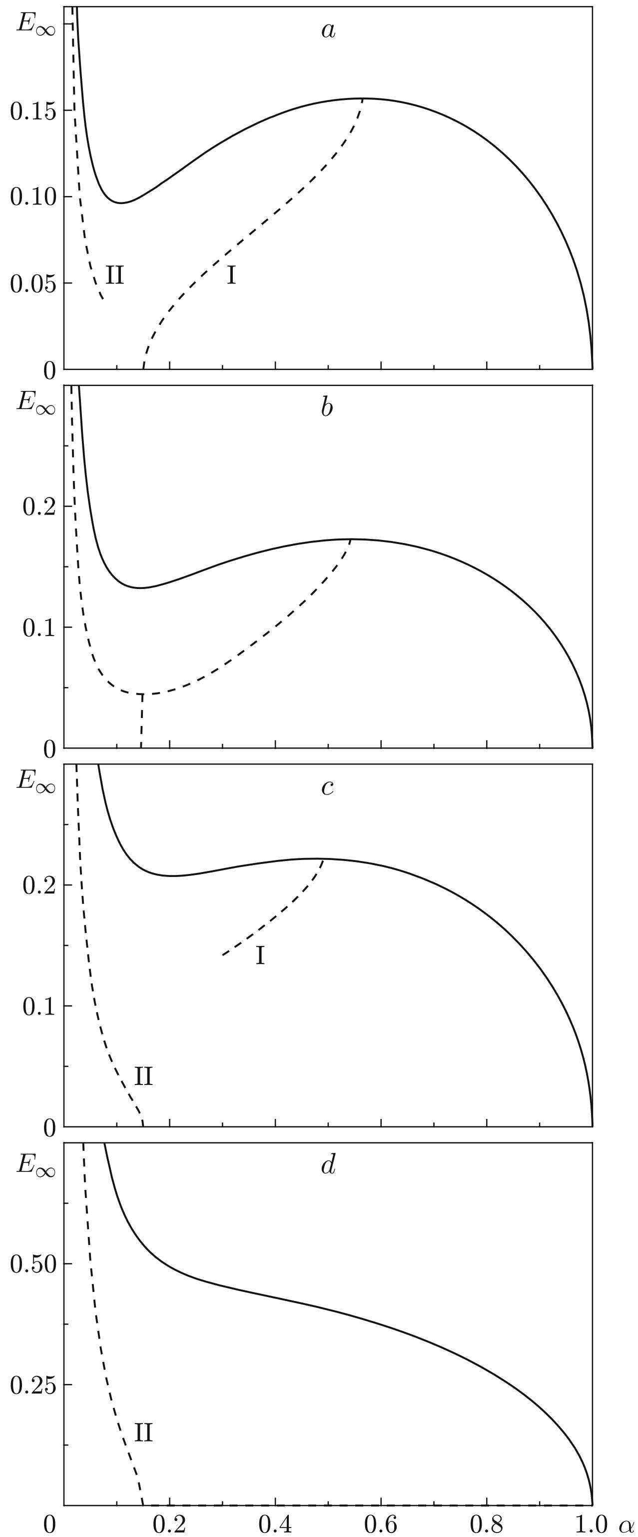

Fig. 3

Fig. 2. Bifurcations of the linear growth parameters for $\mathrm{Re} = 0.1$: (a) $\Omega = 0.2$ ($\Omega < \Omega_1$); (b) $\Omega = 0.33$ ($\Omega = \Omega_1$); (c) $\Omega = 0.4$ ($\Omega_1 < \Omega < \Omega_2$); (d) $\Omega = 1$ ($\Omega > \Omega_2$).

Fig. 3. Maximum growth parameters in the plane (α, E_∞) for $\mathrm{Re} = 0.1$: (a) $\Omega = 0.2$ ($\Omega < \Omega_1$); (b) $\Omega = 0.33$ ($\Omega = \Omega_1$); (c) $\Omega = 0.4$ ($\Omega_1 < \Omega < \Omega_2$); (d) $\Omega = 1$ ($\Omega > \Omega_2$); solid curves are neutral stability curves; dashed curves are maximum growth curves.

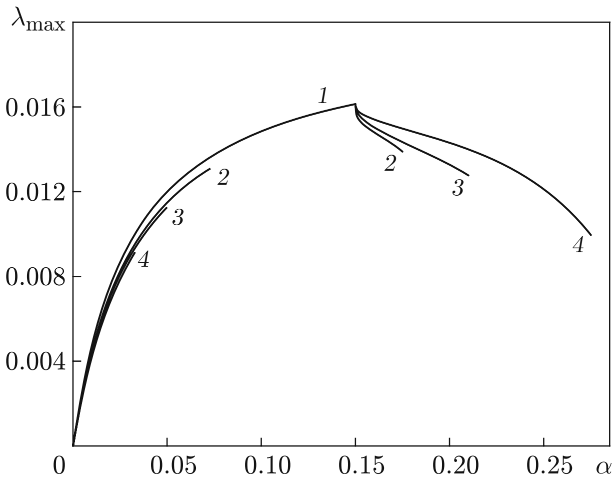

Fig. 4

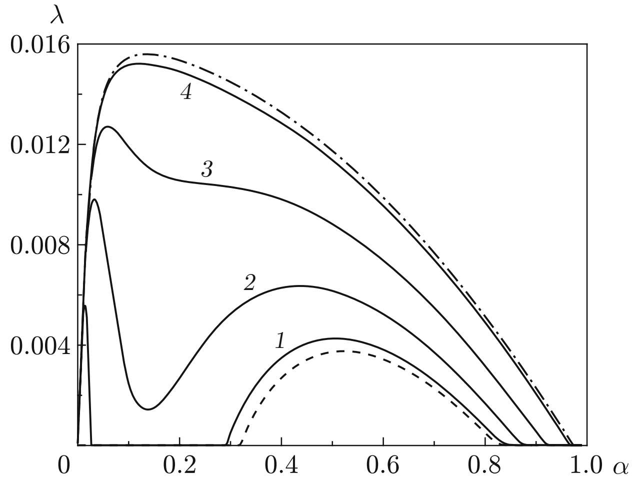

Fig. 5

Fig. 4. Maximum growth parameter $\lambda_{\max}$ versus perturbation wavenumber α for $\Omega = 0.35$ (1), 0.3 (2), 0.25 (3), and 0.2 (4).

Fig. 5. Linear growth parameter versus perturbation wavenumber for $E_\infty = 0.35$, Re = 0.1 and $\Omega = 0.2$ (1), 0.5 (2), 1 (3), and 3 (4); dashed curve refers to $\Omega = 0$; dot-and-dash curve to $\Omega = \infty$.

where $\beta^2 = \alpha^2 + \lambda\,\mathrm{Re}$. The general solution of Eq. (44) which is regular for $y = 0$ is written as

$$\alpha \neq \beta: \qquad \hat{\psi}(y) = i\hat{A}yI_1(\beta y) + i\hat{B}yI_1(\alpha y); \tag{46}$$

$$\alpha = \beta: \qquad \hat{\psi}(y) = i\hat{A}y^2 I_0(\alpha y) + i\hat{B}yI_1(\alpha y). \tag{47}$$

5. Case $\boldsymbol{\alpha \neq \beta}$. Asymptotic Stability. The condition $\alpha \neq \beta$ is equivalent to the fact that $\lambda \neq 0$. Hence, in this case, the perturbations can increase or decrease exponentially. Substituting representation (46) into boundary conditions (45) taking into account the known recursive properties of the Bessel functions, we obtain the following linear homogeneous system of equations for the unknown constants $\hat{A}$, $\hat{B}$, and $\hat{h}$:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{h} \end{pmatrix} = 0. \tag{48}$$

Here

$$a_{11} = 2\alpha^2(I_1(\beta) - \beta I_0(\beta))/\,\mathrm{Re}, \qquad a_{12} = -\alpha\lambda I_0(\alpha) + 2\alpha^2(I_1(\alpha) - \alpha I_0(\alpha))/\,\mathrm{Re},$$

$$a_{13} = \alpha(1 - \alpha^2) + (1 - 1/\delta)2E_\infty^2\alpha^2\Pi_R,$$

$$a_{21} = (\lambda + 2\alpha^2/\,\mathrm{Re})I_1(\beta), \qquad a_{22} = 2\alpha^2 I_1(\alpha)/\,\mathrm{Re}, \qquad a_{23} = (2\delta/\Omega)E_\infty^2\alpha\Pi_I I_1(\alpha)/I_0(\alpha),$$

$$a_{31} = \alpha I_1(\beta), \qquad a_{32} = \alpha I_1(\alpha), \qquad a_{33} = -\lambda.$$

The presence of nontrivial solutions of system (48) is equivalent to the satisfaction of the dispersion relation

$$D(\lambda, \alpha) = \det(a_{ij}) = 0. \tag{49}$$

6. Case $\boldsymbol{\alpha = \beta}$. Neutral Stability. For $\lambda \to 0$, we have $\beta \to \alpha$ and the first two columns of the dispersion matrix (a_{ij}) become linearly dependent. In this case, one should replace the dispersion relation (49) by the relation

$$\frac{\partial D}{\partial \lambda}\Big|_{\lambda=0} = 0$$

or use representation (47) to construct a new dispersion matrix.

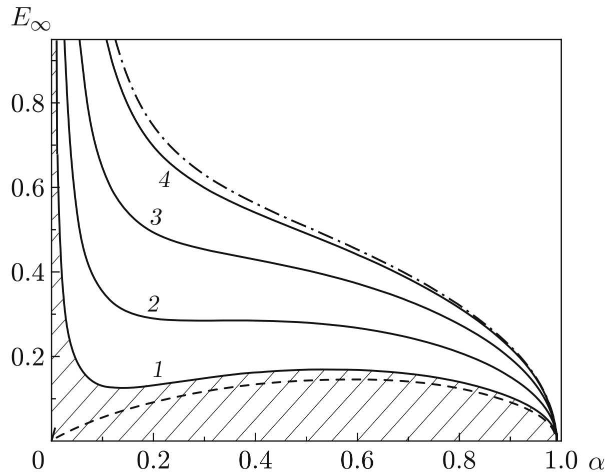

Fig. 6. Field intensity E_∞ versus perturbation wavenumber α for $\Omega = 0.3$ (1), 0.45 (2), 1 (3), and 10 (4); dashed curve refers to $\Omega = 0$; dot-and-dash curve to $\Omega = \infty$; shaded region is the instability region for $\Omega = 0.3$.

The neutral stability problem is written as

$$\frac{2\delta}{\Omega}\, E_\infty^2 \alpha \Pi_I \Big\{ 1 + \alpha\Big(\frac{I_1(\alpha)}{I_0(\alpha)} - \frac{I_0(\alpha)}{I_1(\alpha)}\Big)\Big\} + 2\Big(1 - \frac{1}{\delta}\Big) E_\infty^2 \alpha \Pi_R + 1 - \alpha^2 = 0. \tag{50}$$

The Reynolds number is not explicitly included in (50). Hence, neutral stability is completely determined by the parameters α, δ, E_∞, and Ω.

We note that, for $\Omega = \infty$ (which corresponds to $\omega = \infty$), the neutral stability problem becomes a similar problem for an inviscid liquid [15]. Thus, at high oscillation frequencies, viscosity does not influence the neutral stability. This effect can be considered an analog of the well-known effect of classical mechanics — the disappearance of friction at high-frequency oscillations.

7. Numerical Analysis of Dispersion Relations. The internal parameter of the problem is the perturbation wavenumber α, and the external parameters of the problem are the ratio of the dielectric permittivities δ, the Reynolds number Re, the electric-field intensity E_∞, and the parameter $\Omega = \varepsilon^2 \omega \,\mathrm{Pe}\,/(2\Lambda)$, which is proportional to the dimensionless oscillation frequency of the field ω. All calculations were performed for the value $\delta = 24$, which corresponds to the alcohol used in experiments [9]. The results of calculations for other values of δ, in particular, for the value $\delta = 70$, which corresponds to water, are in qualitative agreement. The calculations show that, for small Reynolds numbers characteristic of microjets, the jet motion depends weakly on the value of Re. All calculations were performed for $\mathrm{Re} = 0.1$.

An analysis of the results of calculations using relations (49) and (50) shows that the main linear-stability characteristics depend significantly on the control parameter Ω. There are two critical values $\Omega_1 \approx 0.33$ and $\Omega_2 \approx 0.45$ for which there is bifurcation of the linear growth parameters λ and the maximum growth curves. Figure 2 gives curves of the linear growth parameters versus perturbation wavenumber for various values of Ω. Figure 3 gives the corresponding neutral stability curves in the plane of the parameters (α, E_∞) (solid curves) and the maximum growth curves (dashed curves). The bifurcation value of Ω_1 depends weakly on the Reynolds number. The value of Ω_2 is determined from the neutral stability problem.

For $\Omega < \Omega_1$, the maximum growth curve has two branches — I and II. The first maximum for $E_\infty = 0$, which arose on branch I, disappears for large values of the intensity E_∞. Thus, for small and large values of E_∞, the linear growth parameters have a single maximum. For intermediate intensity values, two maxima are observed. The maximum of branch I is first larger than the maximum of branch II and then becomes smaller and disappears (see Fig. 3a). For $\Omega < \Omega_1$, there is an interval of wavenumbers in which the maximum growth coefficient is absent (Fig. 4).

For the first critical value $\Omega = \Omega_1 \approx 0.33$, branches I and II of the maximum growth curve merge into one curve (see Fig. 3b) which then again splits into two branches. The corresponding maximum growth parameters for branch I are smaller than the parameters for branch II (see Fig. 3c). For $\Omega = \Omega_2$, the nature of the instability changes. The range of the intensity E_∞ in which there were two intervals of instability is absent.

For $\Omega > \Omega_2$ and any value of E_∞, the linear growth parameters have a single maximum which continuously depends on E_∞. As E_∞ increases, the instability region decreases to zero, and the maximum growth curve has a single continuous branch (see Fig. 3d and Fig. 4). Figure 5 gives a fuller understanding of the nature of the curve of the linear growth parameters versus the parameter Ω.

Figure 6 gives neutral stability curves in the plane of the parameters (α, E_∞) for various values of Ω. It is evident that the external tangential electric field stabilizes the jet. An increase in the intensity E_∞ leads to narrowing of the instability region and a decrease in the linear growth coefficient for wavenumbers for which instability occurs. An increase in the oscillation frequency ω leads to destabilization of the jet due to insignificant narrowing of the stability region in the space of the determining parameters, which, however, is always wider than that in the absence of an electric field. In addition, it follows from the aforesaid that, by increasing the oscillation frequency, one can also completely suppress long-wave instability that arises at low frequencies (for $\Omega < \Omega_1$).

Conclusions. Liquid jets in an external electric field are of interest from the viewpoint of their applications in technological micro- and nanospraying processes and ultrafine fiber manufacture, and from the viewpoint of new types of electrohydrodynamic instability typical of such jets [16, 17]. In particular, the case of a nonstationary external electric field is of great significance.

In the present work, an axisymmetric model was constructed which describes instability of a liquid electrolyte jet placed in an external oscillating electric field determined by two control parameters — the oscillation amplitude and frequency. In a linear approximation, increasing the oscillation amplitude was shown to lead to stabilization of the jet, whereas increasing the frequency leads to insignificant destabilization of the jet. The main linear-stability characteristics were investigated as functions of the external parameters of the problem. It was shown that the examined frequency range contains two bifurcation values upon reaching which the nature of stability changes.

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